Attia, John Okyere. "Semiconductor Physics."
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CHAPTER TEN

SEMICONDUCTOR PHYSICS

In this chapter, a brief description of the basic concepts governing the flow of current in a pn junction are discussed. Both intrinsic and extrinsic semiconductors are discussed. The characteristics of depletion and diffusion capacitance are explored through the use of example problems solved with MATLAB. The effect of doping concentration on the breakdown voltage of pn junctions is examined.

10.1 INTRINSIC SEMICONDUCTORS

10.1.1 Energy bands

According to the planetary model of an isolated atom, the nucleus that contains protons and neutrons constitutes most of the mass of the atom. Electrons surround the nucleus in specific orbits. The electrons are negatively charged and the nucleus is positively charged. If an electron absorbs energy (in the form of a photon), it moves to orbits further from the nucleus. An electron transition from a higher energy orbit to a lower energy orbit emits a photon for a direct band gap semiconductor.

The energy levels of the outer electrons form energy bands. In insulators, the lower energy band (valence band) is completely filled and the next energy band (conduction band) is completely empty. The valence and conduction bands are separated by a forbidden energy gap.
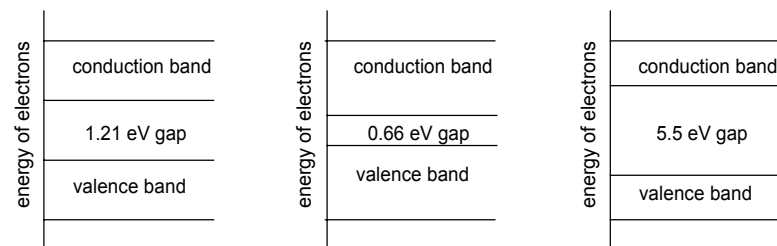

Figure 10.1 Energy Level Diagram of (a) Silicon, (b) Germanium, and (c) Insulator (Carbon)

In conductors, the valence band partially overlaps the conduction band with no forbidden energy gap between the valence and conduction bands. In semiconductors the forbidden gap is less than 1.5eV. Some semiconductor materials are silicon (Si), germanium (Ge), and gallium arsenide (GaAs). Figure 10.1 shows the energy level diagram of silicon, germanium and insulator (carbon).

10.1.2 Mobile carriers

Silicon is the most commonly used semiconductor material in the integrated circuit industry. Silicon has four valence electrons and its atoms are bound together by covalent bonds. At absolute zero temperature the valence band is completely filled with electrons and no current flow can take place. As the temperature of a silicon crystal is raised, there is increased probability of breaking covalent bonds and freeing electrons. The vacancies left by the freed electrons are holes. The process of creating free electron-hole pairs is called ionization. The free electrons move in the conduction band. The average number of carriers (mobile electrons or holes) that exist in an intrinsic semiconductor material may be found from the mass-action law:

$$n_i = AT^{1.5}e^{[-E_g/(kT)]} \qquad (10.1)$$

where

T is the absolute temperature in ^oK

k is Boltzmann's constant (k = 1.38 x 10^{-23} J/K or 8.62x10^{-5} eV/K)

E_g is the width of the forbidden gap in eV. E_g is 1.21 and 1.1eV for Si at 0^oK and 300^oK, respectively. It is given as

$$E_g = E_c - E_v \qquad (10.2)$$

A is a constant dependent on a given material and it is given as

$$A = \frac{2}{h^3}(2\pi m_0 k)^{3/2}(\frac{m_n^* \, m_p^*}{m_0 \, m_o})^{3/4} \qquad (10.3)$$

where

h is Planck's constant (h = 6.62 x 10^{-34} J s or 4.14 x 10^{-15} eV s).

m_o is the rest mass of an electron

m_n* is the effective mass of an electron in a material

m_p* is effective mass of a hole in a material

The mobile carrier concentrations are dependent on the width of the energy gap, E_g, measured with respect to the thermal energy kT.   For small values of T ($kT << E_g$), n_i is small implying, there are less mobile carriers. For silicon, the equilibrium intrinsic concentration at room temperature is

$$n_i = 1.52 \text{ x } 10^{10} \text{ electrons/cm}^3 \tag{10.4}$$

Of the two carriers that we find in semiconductors, the electrons have a higher mobility than holes.   For example, intrinsic silicon at 300^oK has electron mobility of 1350 cm^2 / volt-sec and hole mobility of 480 cm^2 / volt-sec.   The conductivity of an intrinsic semiconductor is given by

$$\sigma_i = q(n_i \mu_n + p_i \mu_p) \tag{10.5}$$

where

q is the electronic charge (1.6 x 10^{-19} C)
n_i is the electron concentration
p_i is the hole concentration. $p_i = n_i$ for the intrinsic semiconductor
μ_n electron mobility in the semiconductor material
μ_p hole mobility in the semiconductor material.

Since electron mobility is about three times that of hole mobility in silicon, the electron current is considerably more than the hole current.   The following example illustrates the dependence of electron concentration on temperature.

Example 10.1

Given that at $T = 300$ °K, the electron concentration in silicon is 1.52×10^{10} electrons $/cm^3$ and $E_g = 1.1$ eV at 300 °K.

(a) Find the constant A of Equation (10.1).
(b) Use MATLAB to plot the electron concentration versus temperature.

Solution

From Equation (10.1), we have

$$1.52x10^{10} = A(300)^{1.5} e^{[-1.1/300*8.62*10^{-5})]}$$

We use MATLAB to solve for A. The width of energy gap with temperature is given as [1].

$$E_g(T) = 1.17 - 4.37x10^{-4}\left(\frac{T^2}{T + 636}\right) \qquad (10.6)$$

Using Equations (10.1) and (10.6), we can calculate the electron concentration at various temperatures.

MATLAB Script

```
%
% Calculation of the constant A
diary ex10_1.dat
k = 8.62e-5;
na = 1.52e10;   ta = 300;
ega = 1.1;
ka  = -ega/(k*ta);
t32a = ta.^1.5;
A = na/(t32a*exp(ka));
fprintf('constant A is %10.5e \n', A)

% Electron Concentration vs. temperature

for i = 1:10
   t(i) = 273 + 10*(i-1);
```

```
   eg(i) = 1.17 - 4.37e-4*(t(i)*t(i))/(t(i) + 636);
   t32(i) = t(i).^1.5;
   ni(i) = A*t32(i)*exp(-eg(i)/(k*t(i)));
end
semilogy(t,ni)
title('Electron Concentration vs. Temperature')
xlabel('Temperature, K')
ylabel('Electron Concentration, cm-3')
```

Result for part (a)

constant A is 8.70225e+024

Figure 10.2 shows the plot of the electron concentration versus temperature.
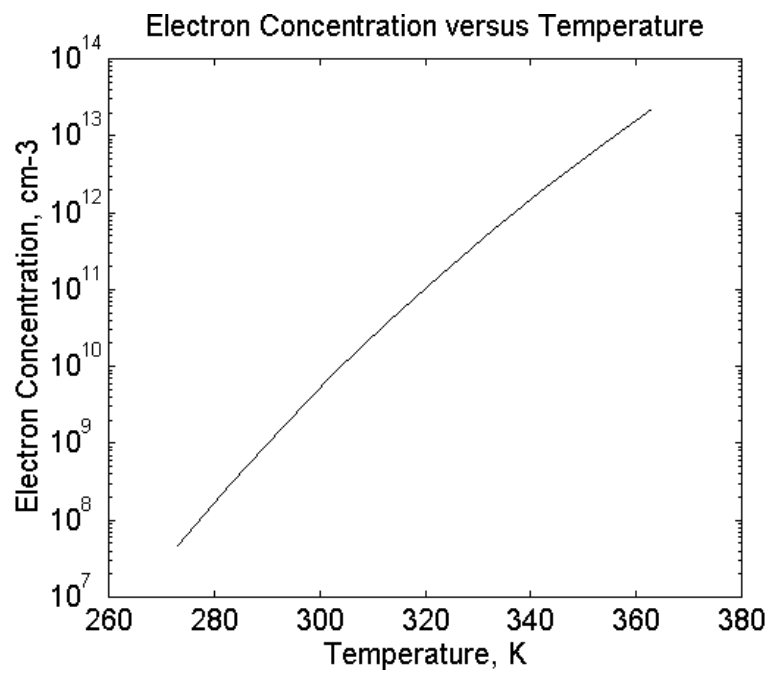

Figure 10.2 Electron Concentration versus Temperature

10.2 EXTRINSIC SEMICONDUCTOR

10.2.1 Electron and hole concentrations

Extrinsic semiconductors are formed by adding specific amounts of impurity atoms to the silicon crystal. An n-type semiconductor is formed by doping the silicon crystal with elements of group V of the periodic table (antimony, arsenic, and phosphorus). The impurity atom is called a donor. The majority carriers are electrons and the minority carriers are holes. A p-type semiconductor is formed by doping the silicon crystal with elements of group III of the periodic table (aluminum, boron, gallium, and indium). The impurity atoms are called acceptor atoms. The majority carriers are holes and minority carriers are electrons.

In a semiconductor material (intrinsic or extrinsic), the law of mass action states that

$$pn = \text{constant} \tag{10.7}$$

where

p is the hole concentration

n is the electron concentration.

For intrinsic semiconductors,

$$p = n = n_i \tag{10.8}$$

and Equation (10.5) becomes

$$pn = n_i^2 \tag{10.9}$$

and n_i is given by Equation (10.1).

The law of mass action enables us to calculate the majority and minority carrier density in an extrinsic semiconductor material. The charge neutrality condition of a semiconductor implies that

$$p + N_D = n + N_A \tag{10.10}$$

where

N_D is the donor concentration

N_A is the acceptor concentration

p is the hole concentration

n is the electron concentration.

In an n-type semiconductor, the donor concentration is greater than the intrinsic electron concentration, i.e., N_D is typically 10^{17} cm^{-3} and $n_i = 1.5$ x 10^{10} cm^{-3} in Si at room temperature. Thus, the majority and minority concentrations are given by

$$n_n \cong N_D \qquad\qquad (10.11)$$

$$p \cong \frac{n_i^2}{N_D} \qquad\qquad (10.12)$$

In a p-type semiconductor, the acceptor concentration N_A is greater than the intrinsic hole concentration $p_i = n_i$. Thus, the majority and minority concentrations are given by

$$p_p \cong N_A \qquad\qquad (10.13)$$

$$n \cong \frac{n_i^2}{N_A} \qquad\qquad (10.14)$$

The following example gives the minority carrier as a function of doping concentration.

Example 10.2

For an n-type semiconductor at 300°K, if the doping concentration is varied from 10^{13} to 10^{18} atoms/cm^3, determine the minority carriers in the doped semiconductors.

Solution

From Equation (10.11) and (10.12),

Electron concentration = N_D and

Hole concentration = $\dfrac{n_i^2}{N_D}$

where

$n_i = 1.5\,2 \text{ x } 10^{10}$ electrons/cm^3

The MATLAB program is as follows:

```
% hole concentration in a n-type semiconductor
nd = logspace(13,18);
n = nd;
ni = 1.52e10;
ni_sq = ni*ni;
p = ni_sq./nd;
semilogx(nd,p,'b')
title('Hole concentration')
xlabel('Doping concentration, cm-3')
ylabel('Hole concentration, cm-3')
```

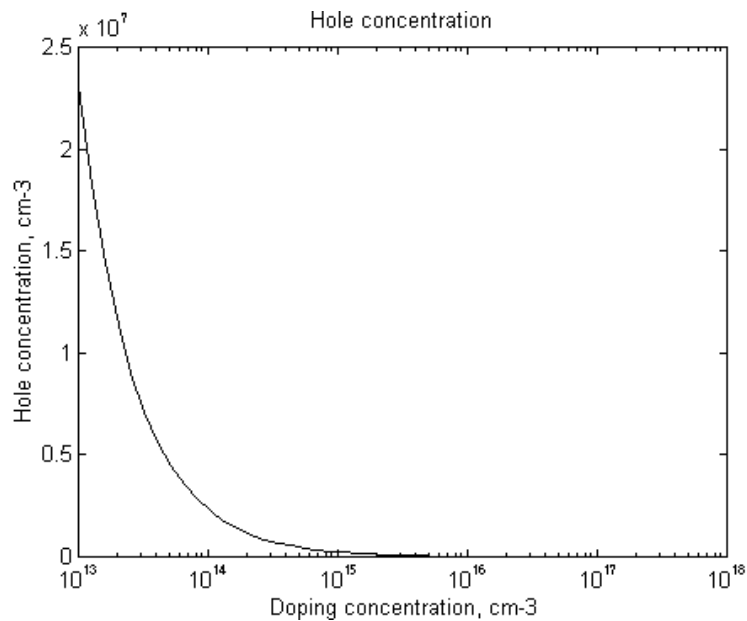Figure 10.3 shows the hole concentration versus doping.

Figure 10.3 Hole Concentration in N-type Semiconductor (Si)

10.2.2 Fermi level

The Fermi level, E_F, is a chemical energy of a material. It is used to describe the energy level of the electronic state at which an electron has the probability of 0.5 occupying that state. It is given as

$$E_F = \frac{1}{2}(E_C + E_V) - \frac{4}{3}KT\ln(\frac{m_n^*}{m_p^*})$$

(10.15)

where

E_C = energy in the conduction band

E_V = energy in the valence band

and k, T, m_n* and m_p* were defined in Section 10.1.

In an intrinsic semiconductor (Si and Ge) m_n* and m_p* are of the same order of magnitude and typically, $E_F \gg kT$. Equation (10.15) simplifies to

$$E_F = E_i \cong \frac{1}{2}(E_C + E_V)$$

(10.16)

Equation (10.16) shows that the Fermi energy occurs near the center of the energy gap in an intrinsic semiconductor. In addition, the Fermi energy can be thought of as the average energy of mobile carriers in a semiconductor material.

In an n-type semiconductor, there is a shift of the Fermi level towards the edge of the conduction band. The upward shift is dependent on how much the doped electron density has exceeded the intrinsic value. The relevant equation is

$$n = n_i e^{[(E_F - E_i)/kT]}$$

(10.17)

where

n is the total electron carrier density

n_i is the intrinsic electron carrier density

E_F is the doped Fermi level

E_i is the intrinsic Fermi level.

In the case of a p-type semiconductor, there is a downward shift in the Fermi level. The total hole density will be given by

$$p = n_i e^{\left[(E_i - E_F)/kT\right]} \tag{10.18}$$

Figure 10.4 shows the energy band diagram of intrinsic and extrinsic semiconductors.

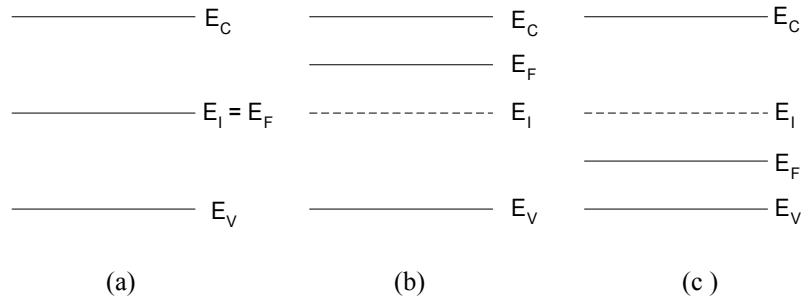|		
———— E_C	———— E_C	———— E_C
	———— E_F	
———— $E_I = E_F$	- - - - - E_I	- - - - - E_I
		———— E_F
———— E_V	———— E_V	———— E_V
(a)	(b)	(c)

Figure 10.4 Energy-band Diagram of (a) Intrinsic, (b) N-type, and (c) P-type Semiconductors.

10.2.3 Current density and mobility

Two mechanisms account for the movement of carriers in a semiconductor material: drift and diffusion. Drift current is caused by the application of an electric field, whereas diffusion current is obtained when there is a net flow of carriers from a region of high concentration to a region of low concentration. The total drift current density in an extrinsic semiconductor material is

$$J = q(n\mu_n + p\mu_p)E \tag{10.19}$$

where

J	is current density
n	is mobile electron density
p	is hole density,
μ_n	is mobility of an electron
μ_p	is mobility of a hole
q	is the electron charge

$$\text{E} \qquad \text{is the electric field.}$$

The total conductivity is

$$\sigma = q(n\mu_n + p\mu_p)\text{E} \tag{10.20}$$

Assuming that there is a diffusion of holes from an area of high concentration to that of low concentration, then the current density of holes in the x-direction is

$$J_p = -qD_p \frac{dp}{dx} \qquad \text{A/cm}^2 \tag{10.21}$$

where

q is the electronic charge

D_p is the hole diffusion constant

p is the hole concentration.

Equation (10.21) also assumes that, although the hole concentration varies along the x-direction, it is constant in the y and z-directions. Similarly, the electron current density, J_n, for diffusion of electrons is

$$J_n = qD_n \frac{dn}{dx} \quad \text{A / cm}^2 \tag{10.22}$$

where

D_n is the electron diffusion constant

n is the electron concentration.

For silicon, D_p = 13 cm^2/s , and D_n = 200 cm^2/s . The diffusion and mobility constants are related, under steady-state conditions, by the Einstein relation

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{kT}{q} \tag{10.23}$$

The following two examples show the effects of doping concentration on mobility and resistivity.

Example 10.3

From measured data, an empirical relationship between electron (μ_n) and hole (μ_p) mobilities versus doping concentration at 300°K is given as [2]

$$\mu_n(N_D) = \frac{5.1x10^{18} + 92\,N_D^{0.91}}{3.75x10^{15} + N_D^{0.91}} \qquad (10.24)$$

$$\mu_{pn}(N_A) = \frac{2.9x10^{15} + 47.7\,N_A^{0.76}}{5.86x10^{12} + N_A^{0.76}} \qquad (10.25)$$

where

N_D and N_A are donor and acceptor concentration per cm^3, respectively.

Plot the μ_n (N_D) and μ_p (N_A) for the doping concentrations from 10^{14} to 10^{20} cm^{-3}.

Solution

MATLAB Script

```
% nc - is doping concentration
%
nc = logspace(14,20);
un = (5.1e18 + 92*nc.^0.91)./(3.75e15 + nc.^0.91);
up = (2.90e15 + 47.7*nc.^0.76)./(5.86e12 + nc.^0.76);
semilogx(nc,un,'w',nc,up,'w')
text(8.0e16,1000,'Electron Mobility')
text(5.0e14,560,'Hole Mobility')
title('Mobility versus Doping')
xlabel('Doping Concentration in cm-3')
ylabel('Bulk Mobility (cm2/v.s)')
```

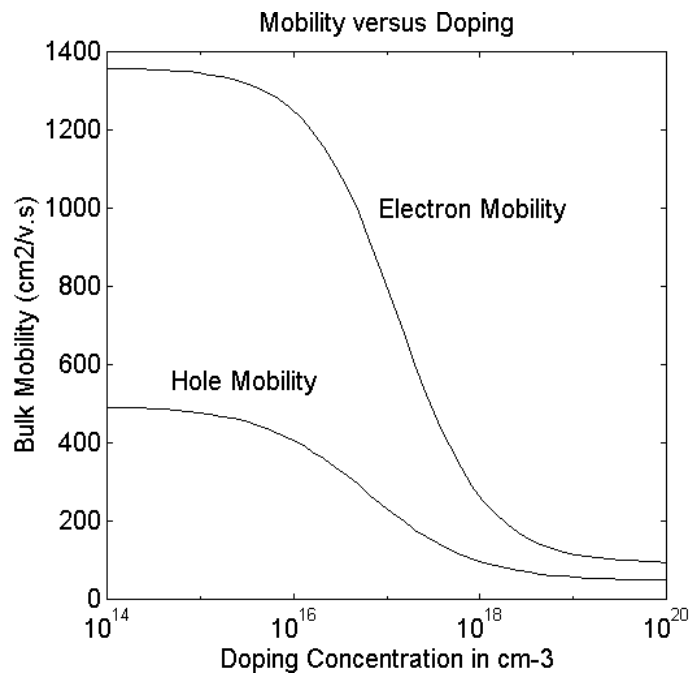Figure 10.5 shows the plot of mobility versus doping concentration.

Figure 10.5 Mobility versus Doping Concentration

Example 10.4

At the temperature of 300°K, the resistivity of silicon doped by phosphorus is given as [3]

$$\rho_n = \frac{3.75x10^{15} + N_D^{0.91}}{1.47x10^{-17}\,N_D^{1.91} + 8.15x10^{-1}\,N_D} \tag{10.26}$$

A similar relation for silicon doped with boron is given as [4]

$$\rho_p = \frac{5.86x10^{12} + N_A^{0.76}}{7.63x10^{-18}\,N_A^{1.76} + 4.64*10^{-4}\,N_A} \tag{10.27}$$

where

N_D and N_A are donor and acceptor concentrations, respectively.

Use MATLAB to plot the resistivity versus doping concentration (cm^{-3}).

Solution

MATLAB Script

```
% nc is doping concentration
% rn - resistivity of n-type
% rp - resistivity of p-type

nc = logspace(14,20);
rn = (3.75e15 + nc.^0.91)./(1.47e-17*nc.^1.91 + 8.15e-1*nc);
rp = (5.86e12 + nc.^0.76)./(7.63e-18*nc.^1.76 + 4.64e-4*nc);

semilogx(nc,rn,'w',nc,rp,'w')
axis([1.0e14, 1.0e17,0,140])
title('Resistivity versus Doping')
ylabel('Resistivity (ohm-cm)')
xlabel('Doping Concentration cm-3')
text(1.1e14,12,'N-type')
text(3.0e14,50,'P-type')
```

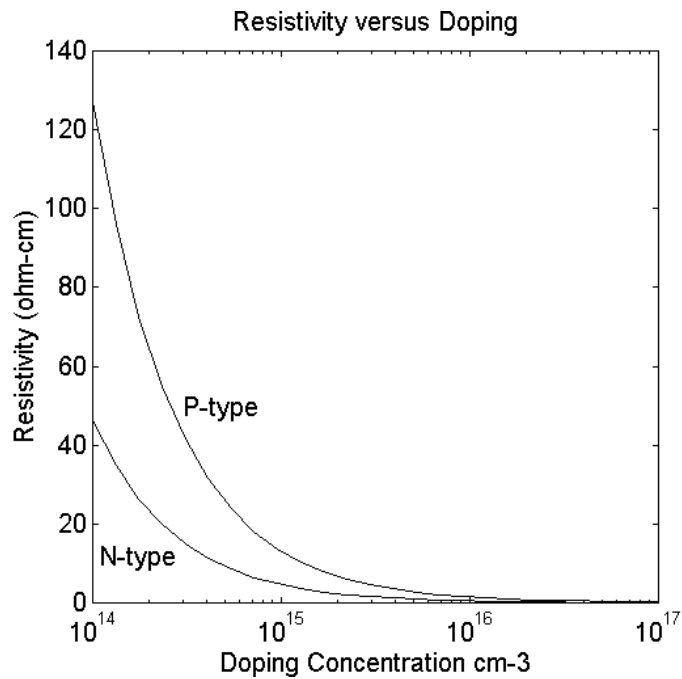Figure 10.6 shows the resistivity of N- and P-type silicon.

Figure 10.6 Resistivity versus Doping Concentration

10.3   PN JUNCTION: CONTACT POTENTIAL, JUNCTION CURRENT

10.3.1   Contact potential

An ideal pn junction is obtained when a uniformly doped p-type material abruptly changes to n-type material. This is shown in Figure 10.7.
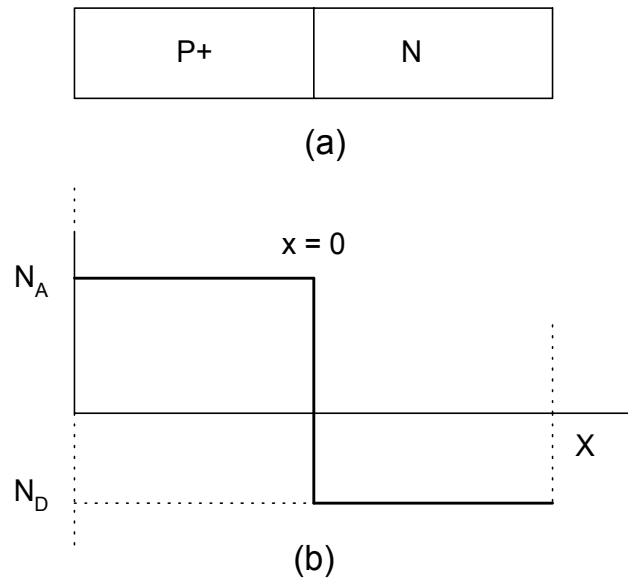
Figure 10.7 Ideal pn Junction (a) Structure, (b) Concentration of Donors (N_D), and acceptor (N_A) impurities.

Practical pn junctions are formed by diffusing into an n-type semiconductor a p-type impurity atom, or vice versa. Because the p-type semiconductor has many free holes and the n-type semiconductor has many free electrons, there is a strong tendency for the holes to diffuse from the p-type to the n-type semiconductors. Similarly, electrons diffuse from the n-type to the p-type material. When holes cross the junction into the n-type material, they recombine with the free electrons in the n-type. Similarly, when electrons cross the junction into the p-type region, they recombine with free holes. In the junction a transition region or depletion region is created.

In the depletion region, the free holes and electrons are many magnitudes lower than holes in p-type material and electrons in the n-type material. As electrons and holes recombine in the transition region, the region near the junction within the n-type semiconductor is left with a net positive charge. The region near the junction within the p-type material will be left with a net negative charge. This is illustrated in Figure 10.8.

Because of the positive and negative fixed ions at the transition region, an electric field is established across the junction. The electric field creates a potential difference across the junction, the potential barrier. The latter is also

called diffusion potential or contact potential, V_C. The potential barrier prevents the flow of majority carriers across the junction under equilibrium conditions.
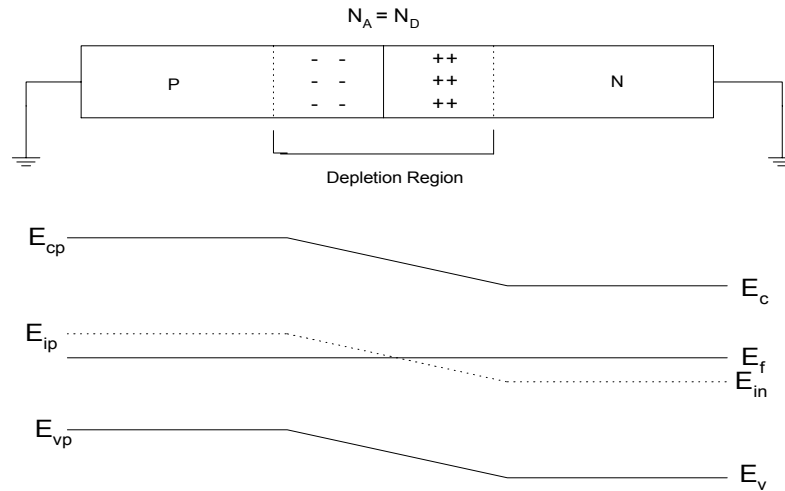
N_A = N_D

Depletion Region

E_{cp}

E_c

E_{ip}

E_f
E_{in}

E_{vp}

E_v

Figure 10.8 pn Junction (a) Depletion region with Positive and Negative Ions (b) Energy Band Diagram near a pn Junction.

The contact potential, V_C, may be obtained from the relations

$$\frac{n_n}{n_p} = e^{\left(\frac{qV_C}{kT}\right)} = \frac{p_p}{p_n}$$ (10.28)

or

$$V_C = \frac{kT}{q}\ln(\frac{n_n}{n_p}) = \frac{kT}{q}\ln(\frac{p_p}{p_n})$$ (10.29)

But, noting that $p_p \cong N_A$, $n_p \cong \dfrac{n_i^2}{N_A}$, $n_n \cong N_D$, $p_n \cong \dfrac{n_i^2}{N_D}$,

Equation (10.29) becomes

$$V_C = \frac{kT}{q}\ln(\frac{N_A N_D}{n_i^2})$$ (10.30)

The contact potential can also be obtained from the band-bending diagram of the pn junction shown in Figure 10.8. That is, from Figure 10.8

$$V_C = \frac{E_{in} - E_{ip}}{q} \tag{10.31}$$

or

$$V_C = -(\phi_{fn} + |\phi_{fp}|) \tag{10.32}$$

where

ϕ_{FN} and ϕ_{FP} are the electron and hole Fermi potentials, respectively. They are given as

$$\phi_{FN} = \frac{E_F - E_{IN}}{q} = \frac{kT}{q} \ln\left(\frac{N_D}{n_i}\right) \tag{10.33}$$

and

$$\phi_{FP} = \frac{E_F - E_{IP}}{q} = \frac{kT}{q} \ln\left(\frac{N_A}{n_i}\right) \tag{10.34}$$

Using Equations (10.31) to (10.34), we have

$$V_C = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right) \tag{10.35}$$

It should be noted that Equations (10.30) and (10.35) are identical. Typically, V_C is from 0.5 to 0.8 V for the silicon pn junction. For germanium, V_C is approximately 0.1 to 0.2, and that for gallium arsenide is 1.5V.

When a positive voltage V_S is applied to the p-side of the junction and n-side is grounded, holes are pushed from the p-type material into the transition region. In addition, electrons are attracted to transition region. The depletion region decreases, and the effective contact potential is reduced. This allows majority carriers to flow through the depletion region. Equation (10.28) modifies to

$$\frac{n_n}{n_p} = e^{\left[\frac{q(V_G - V_S)}{kT}\right]} = \frac{p_p}{p_n} \tag{10.36}$$

When a negative voltage V_S is applied to the p-side of a junction and the n-side is grounded, the applied voltage adds directly to the contact potential. The depletion region increases and it become more difficult for the majority carriers to flow across the junction. The current flow is mainly due to the flow of minority carriers. Equation (10.28) modifies to

$$\frac{n_n}{n_p} = e^{\left[\frac{q(V_C + V_S)}{kT}\right]} = \frac{p_p}{p_n} \tag{10.37}$$

Figure 10.9 shows the potential across the diode when a pn junction is forward-biased and reversed-biased.
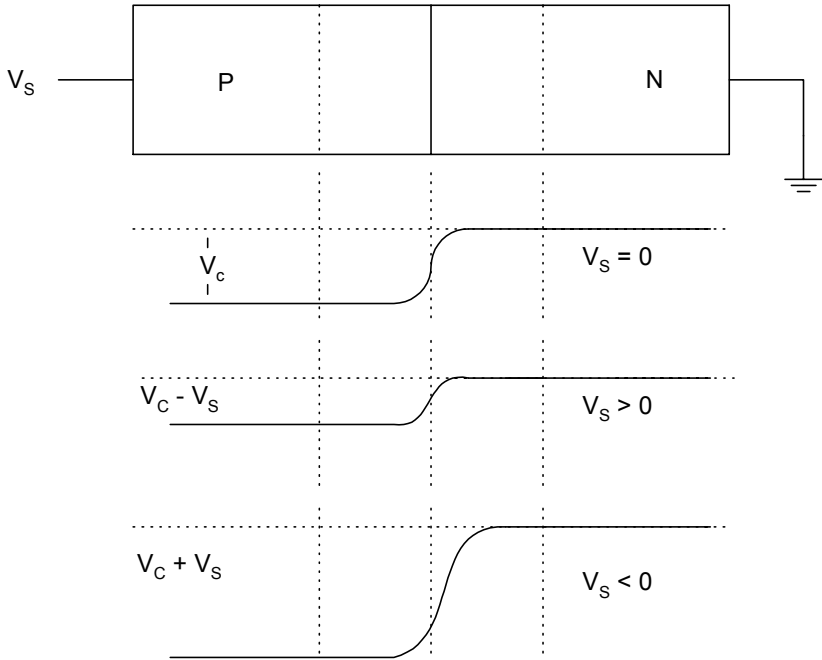

Figure 10.9 PN Junction (a) with Depletion Layer and Source Connection (b) Contact Potential with No Source Voltage (V_S = 0) (c) Junction Potential for Forward-biased pn Junction (V_S > 0) and (d) Junction Potential for Reversed-biased pn Junction (V_S < 0)

The following example illustrates the effect of source voltage on the junction potential.

Example 10.5

For a Silicon pn junction with $N_D = 10^{14}$ cm^{-3} and $N_A = 10^{17}$ cm^{-3} and with $n_i^2 = 1.04 \times 10^{26}$ cm^{-6} at $T = 300$ °K,

(a)   Calculate the contact potential.

(b)   Plot the junction potential when the source voltage V_S of Figure 10.9 increases from -1.0 to 0.7 V.

Solution

MATLAB Script

```
diary ex10_5.dat
% Junction potential versus source voltage
% using equation(10.36) contact potential is

t = 300;
na = 1.0e17;
nd = 1.0e14;
nisq = 1.04e20;
q = 1.602e-19;
k = 1.38e-23;

% calculate contact potential
vc = (k*t/q)*(log(na*nd/nisq))
vs = -1.0:0.1:0.7;
jct_pot = vc - vs;

% plot curve
plot(vs,jct_pot)
title('Junction potential vs. source voltage')
xlabel('Source voltage, V')
ylabel('Junction potential, V')
diary
```

(a) The contact potential is

vc =
        0.6535

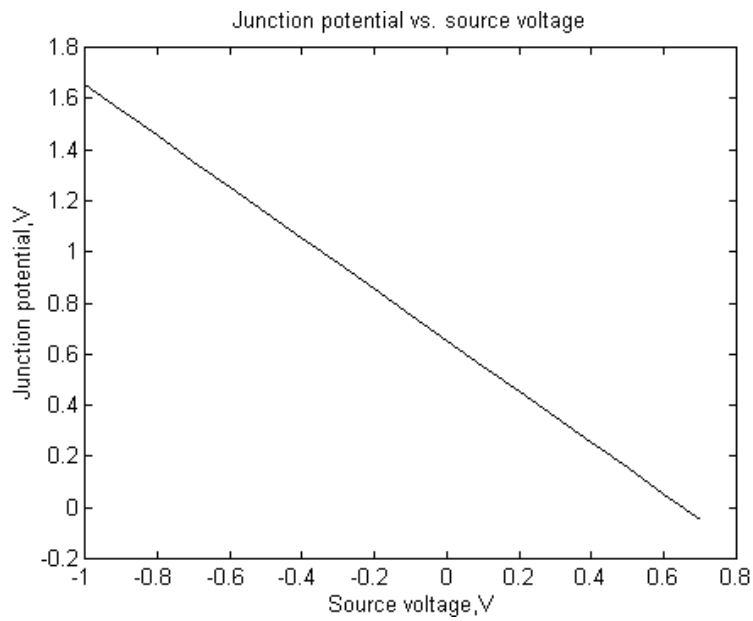(b) Figure 10.10 shows the graph of the junction potential versus the source voltage.

Figure 10.10 Junction Potential versus Source Voltage.

10.3.2 Junction current

The pn junction current is given as

$$I = I_s \left[e^{\left(\frac{qV_S}{kT} \right)} - 1 \right] \tag{10.38}$$

where

V_S is the voltage across the pn junction [see Figure 10.9 (a)]

q is the electronic charge

T is the absolute temperature

k is Boltzmann's constant

I_S is reverse saturation current. It is given as

$$I_S = qA(\frac{D_p p_n}{L_p} + \frac{D_n n_p}{L_n})$$ (10.39)

where

A	is the diode cross-sectional area
L_p, L_n	are the hole and electron diffusion lengths
p_n, n_p	are the equilibrium minority carrier concentrations
D_p, D_n	are the hole and electron diffusion coefficients, respectively.

Since $p_n \cong \dfrac{n_i^2}{N_D}$ and $n_p \cong \dfrac{n_i^2}{N_A}$, Equation (10.39) becomes

$$I_S = qA\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right)n_i^2$$ (10.40)

The diffusion coefficient and diffusion length are related by the expression

$$L_p = \sqrt{D_p \tau_p}$$ (10.41)

and

$$L_n = \sqrt{D_n \tau_n}$$ (10.42)

where

τ_p, τ_n are the hole minority and electron minority carrier lifetime, respectively.

Equation (10.38) is the diode equation. It is applicable for forward-biased ($V_S > 0$) and reversed-biased ($V_S < 0$) pn junctions.

Using Equations (10.1) and (10.39), the reverse saturation current can be rewritten as

$$I_S = k_1 T^3 e^{\left[-E_g/(kT)\right]} \tag{10.43}$$

where k_1 is a proportionality constant

$$\frac{dI_S}{dT} = 3 k_1 T^2 e^{-\frac{E_g}{kT}} + k_1 T^3 \frac{-E_g}{kT^2} e^{-\frac{E_g}{kT}}$$

Thus

$$\frac{1}{I_S}\frac{dI_S}{dT} = \frac{3}{T} + \frac{1}{T}\frac{E_g}{kT} = \frac{3}{T} + \frac{1}{T}\frac{V_g}{V_T} \tag{10.44}$$

where

$$V_T = \frac{kT}{q} \qquad \text{and} \qquad V_g = \frac{E_g}{q}$$

For silicon at room temperature,

$$\frac{V_g}{V_T} = 44.4.$$

Thus

$$\frac{dI_S}{dT} = (3 + \frac{V_g}{V_T})\frac{dT}{T} = 47.4\frac{dT}{T} \tag{10.45}$$

At room temperature (300° K), the saturation current approximately doubles every 5° C [5]. The following example shows how I_S is affected by temperature.

Example 10.6

A silicon diode has $I_S = 10^{-15}$ A at 25° C and assuming I_S increases by 15% per °C rise in temperature, find and plot the value of I_S from 25 °C to 125 °C.

Solution

From the information given above, the reverse saturation current can be expressed as

$$I_S = 10^{-15}(1.15)^{(T-25)}$$

MATLAB is used to find I_S at various temperatures.

MATLAB Script

```
% Saturation current
%

t = 25:5:125;
is = 1.0e-15*(1.15).^(t-25);

plot(t,is)
title('Reverse Saturation Current vs. Temperature')
xlabel('Temperature, C')
ylabel('Current, A')
```

Figure 10.11 shows the effect of temperature on the reverse saturation current.
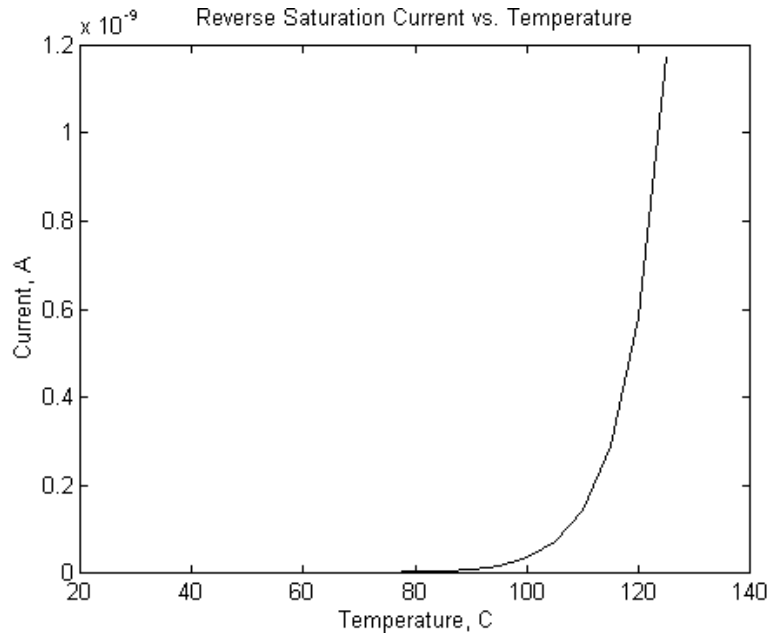
Figure 10.11 Reverse Saturation Current versus Temperature

10.4 DEPLETION AND DIFFUSION CAPACITANCES

10.4.1 Depletion capacitance

As mentioned previously, a pn junction is formed when a p-type material is joined to an n-type region. During device fabrication, a p-n junction can be formed using process such as ion-implantation diffusion or epitaxy. The doping profile at the junction can take several shapes. Two popular doping profiles are abrupt (step) junction and linearly graded junction.

In the abrupt junction, the doping of the depletion region on either side of the metallurgical junction is a constant. This gives rise to constant charge densities on either side of the junction. This is shown in Figure 10.12.
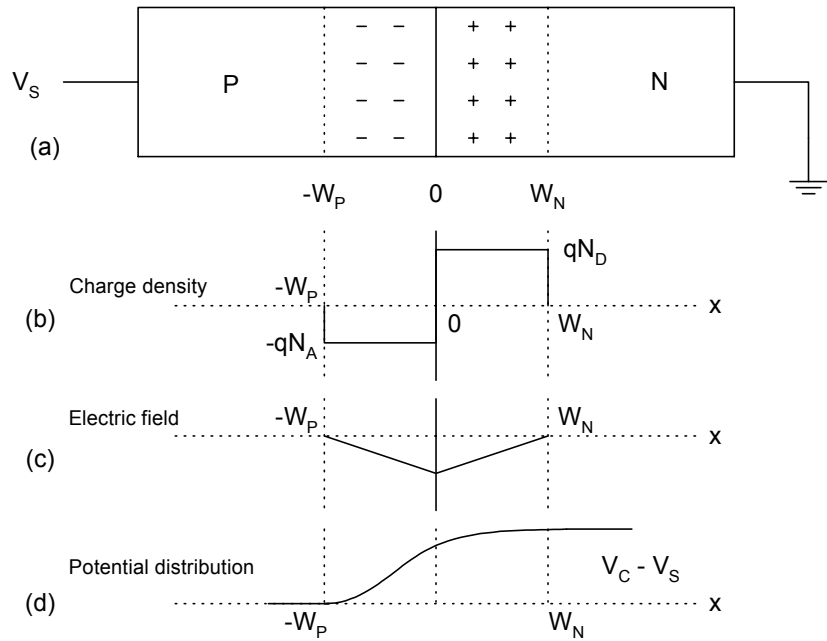
Figure 10.12   PN Junction with Abrupt Junction (a) Depletion Region (b) Charge Density (c) Electric Field and (d) Potential Distribution

For charge equality,

$$qN_A W_P = qN_D W_N \tag{10.46}$$

it can be shown [6] that the depletion width in the p-type (W_P) and that of the n-type material (W_N) can be given as

$$W_P = \sqrt{\frac{2\varepsilon N_D (V_C - V_s)}{qN_A (N_D + N_A)}} \tag{10.47}$$

$$W_N = \sqrt{\frac{2\varepsilon N_A (V_C - V_s)}{q N_D (N_D + N_A)}} \qquad (10.48)$$

where

ε is the relative dielectric constant

       ($\varepsilon = 12\varepsilon_0$ for Si, and ε_0 = 8.85 x 10^{-12} F/m)

N_D is donor concentration

N_A is acceptor concentration

q is electronic charge

V_C is contact potential obtained from Equation (10.30)

V_S is source voltage.

If the doping density on one side of the metallurgical junction is greater than that on the other side (i.e., $N_A \gg N_D$ or $N_D \gg N_A$), then the junction properties are controlled entirely by the lightly doped side. This condition is termed the one-sided step junction approximation. This is the practical model for shallow junctions formed by a heavily doped diffusion into a lightly doped region of opposite polarity [7].

In a linearly graded junction, the ionized doping charge density varies linearly across the depletion region. The charge density passes through zero at the metallurgical junction. Figure 10.13 shows the profile of the linearly graded junction.
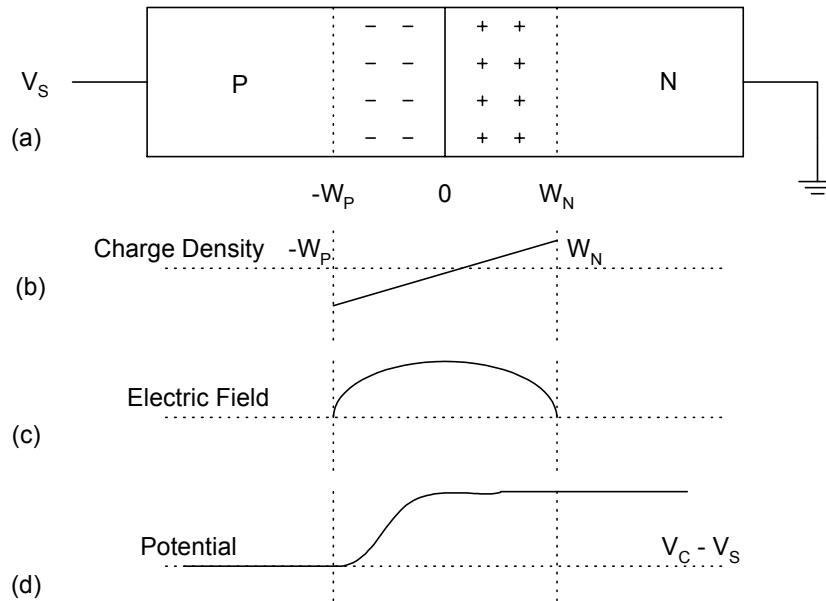
Figure 10.13 PN Junction with Linearly Graded Junction
(a) Depletion Region (b) Charge Density
(c) Electric Field (d) Potential Distribution

For a linearly graded junction, the depletion width in the p-type and n-type material, on either side of the metallurgical junction, can be shown to be

$$W_N = \left| W_P \right| = \left[\frac{12\varepsilon(V_C - V_S)}{qa} \right]^{\frac{1}{3}}$$

(10.49)

where

a is the slope of the graded junction impurity profile.

The contact potential is given as [6]

$$V_C = \frac{kT}{q} \ln\left(\frac{aW_N}{2n_i}\right)$$

(10.50)

The depletion capacitance, C_j, is due to the charge stored in the depletion region. It is generally given as

$$C_j = \frac{\varepsilon A}{W_T} \tag{10.51}$$

where

$$W_T = W_N + \left| W_P \right| \tag{10.52}$$

A is cross-sectional area of the pn junction.

For abrupt junction, the depletion capacitance is given as

$$C_j = A\sqrt{\frac{\varepsilon q N_A N_D}{2(N_D + N_A)(V_C - V_S)}} \tag{10.53}$$

For linearly graded junction, the depletion capacitance is given as

$$C_j = 0.436(aq)^{1/3} \varepsilon^{2/3} A(V_C - V_S)^{-1/3}$$

$$C_j = 0.436A[\frac{aq\varepsilon^2}{(V_C - V_S)}]^{1/3} \tag{10.54}$$

In general, we may express the depletion capacitance of a pn junction by

$$C_j = \frac{C_{j0}}{\left[1 - \dfrac{V_S}{V_C}\right]^m} \qquad \frac{1}{3} \le m \le \frac{1}{2} \tag{10.55}$$

where

$$m = \frac{1}{3} \quad \text{for linearly graded junction and}$$

$$m = \frac{1}{2} \quad \text{for step junction}$$

C_{j0} = zero-biased junction capacitance. It can be obtained from Equations (10.53) and (10.54) by setting V_S equal to zero.

Equations (10.53 to 10.55) are, strictly speaking, valid under the conditions of reversed-biased $V_S < 0$. The equations can, however, be used when $V_S < 0.2$V. The positive voltage, V_C, is the contact potential of the pn junction. As the pn junction becomes more reversed biased ($V_S < 0$), the depletion capacitance decreases. However, when the pn junction becomes slightly forward biased, the capacitance increases rapidly. This is illustrated by the following example.

Example 10.7

For a certain pn junction, with contact potential 0.065V, the junction capacitance is 4.5 pF for V_S = -10 and C_j is 6.5 pF for V_S = -2 V.

(a) Find m and C_{j0} of Equation (10.55).
(b) Use MATLAB to plot the depletion capacitance from -30V to 0.4V.

Solution

From Equation (10.55)

$$C_{j1} = \frac{C_{j0}}{[1 - \dfrac{V_{S1}}{V_C}]^m}$$

$$C_{j2} = \frac{C_{j0}}{[1 - \dfrac{V_{S2}}{V_C}]^m}$$

therefore

$$\frac{C_{j1}}{C_{j2}} = \left[\frac{V_C - V_{S2}}{V_C - V_{S1}} \right]^m$$

$$m = \frac{\log_{10}\left[\dfrac{C_{j1}}{C_{j2}}\right]}{\log_{10}\left[\dfrac{V_C - V_{S2}}{V_C - V_{S1}}\right]} \tag{10.56}$$

and

$$C_{j0} = C_{j1}\left[1 - \frac{V_{S1}}{V_C}\right]^m \tag{10.57}$$

MATLAB is used to find m and C_{j0}. It is also used to plot the depletion capacitance.

MATLAB Script

```
% depletion capacitance
%
cj1 = 4.5e-12; vs1 = -10;
cj2 = 6.5e-12; vs2 = -2;
vc = 0.65;

num = cj1/cj2;
den = (vc-vs2)/(vc-vs1);
m = log10(num)/log10(den);
cj0 = cj1*(1 - (vs1/vc))^m;
vs = -30:0.2:0.4;
k = length(vs);
for i = 1:k
 cj(i) = cj0/(1-(vs(i)/vc))^m;
end
plot(vs,cj,'w')
xlabel('Voltage,V')
ylabel('Capacitance,F')
title('Depletion Capacitance vs. Voltage')
axis([-30,2,1e-12,14e-12])
```

(a) The values of m, C_{j0} are

```
m  =
      0.02644
```

cj0 =

    9.4246e-012

(b) Figure 10.14 shows the depletion capacitance versus the voltage across the junction.
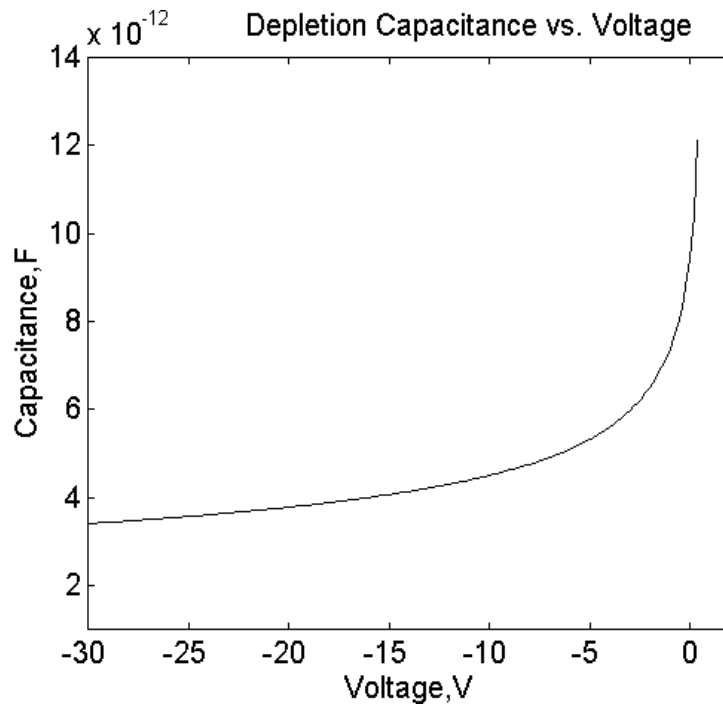

Figure 10.14 Depletion Capacitance of a pn Junction

10.4.2 Diffusion capacitance

When a pn junction is forward biased, holes are injected from the p-side of the metallurgical junction into the n-type material. The holes are momentarily stored in the n-type material before they recombine with the majority carriers (electrons) in the n-type material. Similarly, electrons are injected into and temporarily stored in the p-type material. The electrons then recombine with the majority carriers (holes) in the p-type material. The diffusion capacitance, C_d, is due to the buildup of minority carriers charge around the metallurgical

junction as the result of forward biasing the pn junction. Changing the forward current or forward voltage, ΔV, will result in the change in the value of the stored charge ΔQ, the diffusion capacitance, C_d, can be found from the general expression

$$C_d = \frac{\Delta Q}{\Delta V} \tag{10.58}$$

It turns out that the diffusion capacitance is proportional to the forward-biased current. That is

$$C_d = K_d I_{DF} \tag{10.59}$$

where

$\quad K_d \quad$ is constant at a given temperature

$\quad I_{DF} \quad$ is forward-biased diode current.

The diffusion capacitance is usually larger than the depletion capacitance [1, 6]. Typical values of C_d ranges from 80 to 1000 pF.

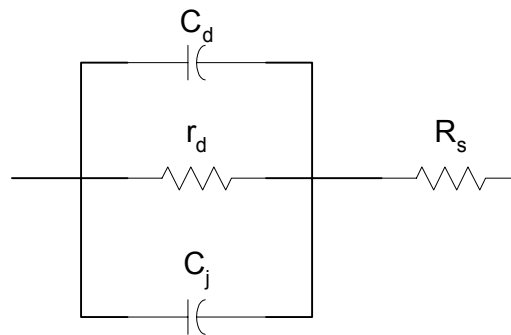A small signal model of the diode is shown in Figure 10.15.

Figure 10.15   Small-signal Model of a Forward-biased pn Junction

In Figure 10.15, C_d and C_j are the diffusion and depletion capacitance, respectively. R_S is the semiconductor bulk and contact resistance. The dynamic resistance, r_d, of the diode is given as

$$r_d = \frac{nkT}{qI_{DF}} \qquad (10.60)$$

where

n	is constant
k	is Boltzmann's constant
T	is temperature in degree Kelvin
q	is electronic charge.

When a pn junction is reversed biased, $C_d = 0$. The model of the diode is shown in Figure 10.16.
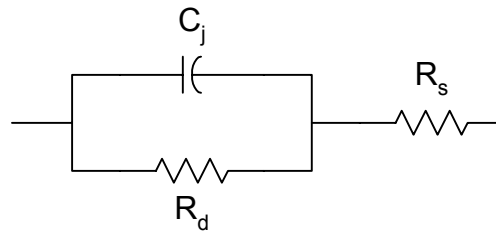

Figure 10.16 Model of a Reverse-biased pn Junction

In Figure 10.16, C_j is the depletion capacitance. The diffusion capacitance is zero. The resistance R_d is reverse resistance of the pn junction (normally in the mega-ohms range).

Example 10.8

A certain diode has contact potential; $V_C = 0.55V$, $C_{j0} = $ diffusion capacitance at zero biased is 8 pF; the diffusion capacitance at 1mA is 100 pF. Use MATLAB to plot the diffusion and depletion capacitance for forward- biased voltages from 0.0 to 0.7 V. Assume that $I_S = 10^{-14}$ A, $n = 2.0$ and step-junction profile.

Solution

Using Equations (10.38) and (10.59), we write the MATLAB program to obtain the diffusion and depletion capacitance.

MATLAB Script

```
%
% Diffusion and depletion Capacitance
%
cd1 = 100e-12; id1 = 1.0e-3; cj0 = 8e-12; vc =0.55;
m = 0.5;
is = 1.0e-14; nd = 2.0;
k = 1.38e-23; q = 1.6e-19; T = 300;
kd = cd1/id1;
vt = k*T/q;
v = 0.0:0.05:0.55;
nv = length(v);

for i = 1:nv
 id(i) = is*exp(v(i)/(nd*vt));
 cd(i) = kd*id(i);
 ra(i) = v(i)/vc;
 cj(i) = cj0/((1 - ra(i)).^m);
end

subplot(121)
plot(v,cd)
title('Diffusion Cap.')
xlabel('Voltage, V'), ylabel('Capacitance, F')

subplot(122)
plot(v,cj)
title('Depletion Cap.')
xlabel('Voltage, V'), ylabel('Capacitance, F')
```

Figure 10.17 shows the depletion and diffusion capacitance of a forward-biased pn junction.
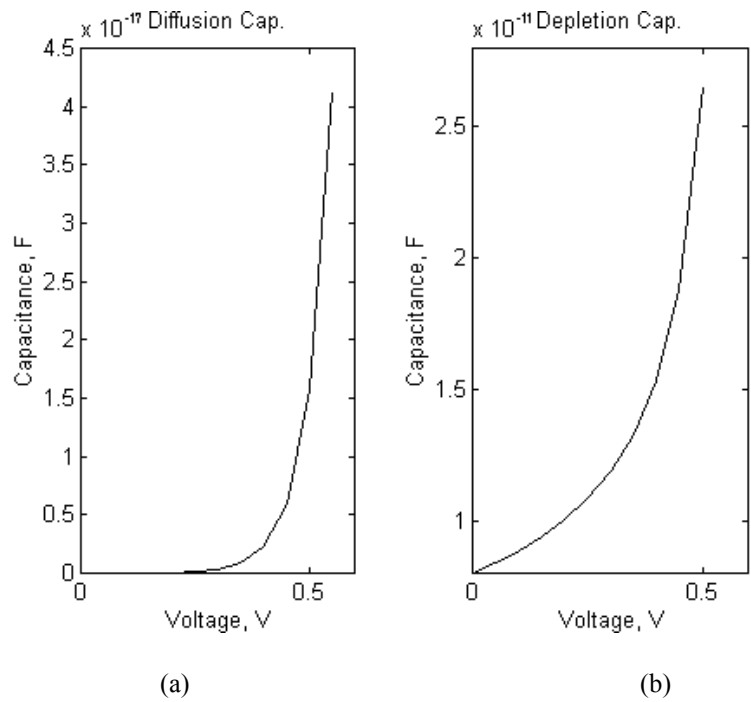
Figure 10.17 (a) Depletion and (b) Diffusion Capacitance

10.5   BREAKDOWN VOLTAGES OF PN JUNCTIONS

The electric field E is related to the charge density through the Poisson's equation

$$\frac{dE(x)}{dx} = \frac{\rho(x)}{\varepsilon_S \varepsilon_0}$$

(10.61)

where

ε_S is the semiconductor dielectric constant

ε_0 is the permittivity of free space, $8.86 * 10^{-14}$ F/cm

$\rho(x)$ is the charge density.

For an abrupt junction with charge density shown in Figure 10.12, the charge density

$$\rho(x) = -qN_A \qquad -W_P < x < 0$$
$$= qN_D \qquad\qquad 0 < x < W_N \tag{10.62}$$

The maximum electric field

$$\left|E_{\max}\right| = \frac{qN_A W_P}{\varepsilon_s \varepsilon_0} = \frac{qN_D W_N}{\varepsilon_s \varepsilon_0} \tag{10.63}$$

Using Equation (10.47) or (10.48, Equation (10.63) becomes

$$\left|E_{\max}\right| = \sqrt{\frac{2qN_D N_A (V_C - V_S)}{\varepsilon_S \varepsilon_0 (N_A + N_D)}} \tag{10.64}$$

For a linearly graded junction, the charge density, $\rho(x)$ is given as (see Figure 10.13)

$$\rho(x) = ax \qquad\qquad -\frac{W}{2} < x < \frac{W}{2} \tag{10.65}$$

and the maximum electric field can be shown to be

$$\left|E_{\max}\right| = \frac{aq}{8\varepsilon_S \varepsilon_0} W^2 \tag{10.66}$$

where

  a is slope of charge density
  W is width of depletion layer and

$$\frac{W}{2} = W_N = W_P$$

The width of the depletion region, W, can be obtained from Equation (10.49).

Equation (10.64) indicates that as the reverse voltage increases, the magnitude of the electric field increases. The large electric field accelerates the carriers

crossing the junction. At a critical field, E_{crit}, the accelerated carriers in the depletion region have sufficient energy to create new electron-hole pairs as they collide with other atoms. The secondary electrons can in turn create more carriers in the depletion region. This is termed the avalanche breakdown process. For silicon with an impurity concentration of 10^{16} cm^{-3}, the critical electric field is about 2.0×10^5 V/cm.

In a highly doped pn junctions, where the impurity concentration is about 10^{18} cm^{-3}, the critical electric field is about 10^6 V/cm. This high electric field is able to strip electrons away from the outer orbit of the silicon atoms, thus creating hole-electron pairs in the depletion region. This mechanism of breakdown is called zener breakdown. This breakdown mechanism does not involve any multiplication effect. Normally, when the breakdown voltage is less than 6V, the mechanism is zener breakdown process. For breakdown voltages beyond 6V, the mechanism is generally an avalanche breakdown process.

For an abrupt junction, where one side is heavily doped, the electrical properties of the junction are determined by the lightly doped side. Experimentally, the breakdown voltage of semiconductor step junction (n$^+$p or p$^+$n) as the function of doping concentration in the lightly doped side is given as [7]

$$V_{BR} = k \left[\frac{N_B}{10^{16}} \right]^{-0.75} \tag{10.67}$$

where

$$k \quad = 25\text{V for Ge}$$
$$= 60\text{V for Si}$$

and

$\quad N_B \quad$ is the doping concentration of lightly doped side.

The following example shows the effect of doping concentration on breakdown voltage.

Example 10.9

Use MATLAB to plot the breakdown voltage versus doping concentration for a one-sided step junction for silicon and germanium, and using doping concentration from 10^{14} to 10^{19} cm^{-3}.

Solution

Using Equation (10.67), we calculate the breakdown voltage for various doping concentration.

MATLAB Script

```
%
% Breakdown voltage
%
k1 = 25;
 k2 = 60;
nb = logspace(14,19);
n = length(nb);

for i = 1:n
 vbr1(i) = k1*(nb(i)/1.0e16)^(-0.75); % Ge breakdown voltage
 vbr2(i) = k2*(nb(i)/1.0e16)^(-0.75); % Si breakdown voltage
end

semilogx(nb,vbr1,'w', nb,vbr2,'w')
xlabel('Impurity Concentration, cm-3')
ylabel('Breakdown Voltage,V')
title('Breakdown Voltage vs. Impurity Concentration')
axis([1.0e14,1.0e17,0,2000])
text(2.0e14,270,'Ge')
text(3.0e14,1000,'Si')
```

Figure 10.18 shows the plot of breakdown voltage of one-sided abrupt junction.
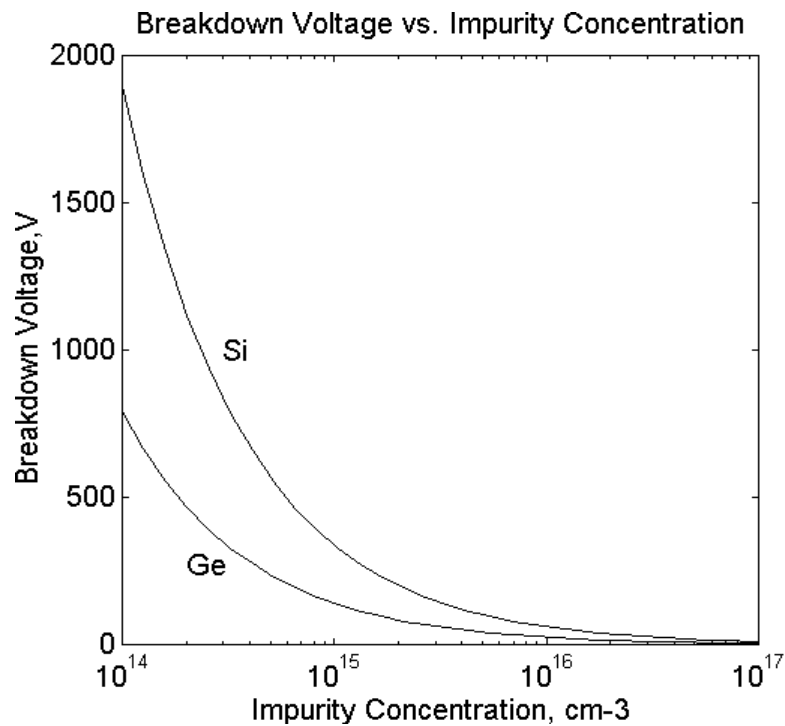
Figure 10.18 Breakdown Voltage versus Impurity Concentration

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EXERCISES

10.1 In the case of silicon for temperature below 700 $^{\circ}$K, the density of intrinsic created carriers, n_i, can be approximated as [8]

$$n_i = 3.87 * 10^{16}\, T^{3/2} e^{-\left(\frac{7.02*10^3}{T}\right)}$$

(10.68)

(a) Use MATLAB to plot the intrinsic carrier concentration versus $(1000/T)$ where T is temperature in degrees Kelvin.

(b) Compare the above relation for intrinsic concentration with that of Example 10.1. Plot the difference between of n_i for Equations (10.1) and (10.68).

10.2 Assuming that at 300 $^{\circ}$K the mobile carrier concentrations of intrinsic germanium and silicon semiconductor materials are $2.390*10^{13}$ and $1.52*10^{10}$, respectively, use MATLAB to plot the $E_F - E_i$ versus donor concentration for Ge and Si. Assume donor concentrations from 10^{10} to 10^{18}.

10.3 For power devices with breakdown voltages above 100V and resistivities greater than 1 ohm-cm (n-type silicon) and 3 ohm-cm (p-type silicon), the resistivity versus doping concentrations can be simplified to

$$\rho_n = 4.596 * 10^{15} \, N_D^{-1}$$

$$\rho_{pn} = 1.263 * 10^{16} \, N_A^{-1}$$

(a) Use MATLAB to plot resistivity versus doping concentration (from 10^{12} to 10^{18} cm^{-3}).
(b) Compare your results with those obtained in Example 10.4.

10.4 For Ge pn junction with $N_A = 10^{18}$ cm^{-3}, $N_D = 10^{15}$ cm^{-3} and n_i at 300 °K is 2.39*10^{13},

(a) Calculate the contact potential.
(b) Plot the junction potential for source voltages of -1.0V to 0.3V.

10.5 For the small signal model of the forward-biased pn junction, shown in Figure 10.15, $R_S = 5\Omega$, $r_d = 10 \, \Omega$, $C_d = 110$ pF at I_{DF} of 1 mA. Use MATLAB to plot the equivalent input impedance (magnitude and phase) for frequencies from 10^4 to 10^{10} Hz.

10.6 Empirically, the breakdown voltage of a linearly graded junction can be approximated as [9]

$$V_{BR} = k \left[\frac{a^{-4}}{10^{21}} \right]^{-0.75}$$

where $k = 18$ V for Ge or 40 V for Si.

Use MATLAB to plot the breakdown voltage vs. impurity gradient of Ge and Si. Use impurity gradient values from 10^{19} to 10^{24}.